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TITLE:

(6) Integral equations for the harmonics on the surface of a body of revolution

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(5) TRANS. FROM  
Moscow. Universitet. Vestnik. Seriya III. Fizika,  
astronomiya, no. 6, 1962 (11-19)

TEXT: When any wave  $u_0(M)$  hits the closed surface  $S$  of a body of revolution on which  $r = f(z)$ ,  $a \leq z \leq b$ , the total field will be  $u(M) = u_0(M) + v(M)$ .  $v(M)$  is determined by the boundary value problem  $\Delta v + k^2 v = 0$ ,  $v|_S = u_0|_S$ ,

$\frac{\partial v}{\partial R} + i k v = O(1/R)$  for  $R \rightarrow \infty$ . Other boundary conditions can be treated in a similar way. The function  $v(M)$  is determined by its values on  $S$  and by its derivatives in the directions of the normal to the surface by means of Green's formula. The author generalizes a method established by N. N. Govorun (DAN SSSR, 126, no. 1, 49, 1959; 132, no. 1, 91, 1960) to obtain a first-kind Fredholm integral equation for the determination of

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$v(M)$ . This equation has a kernel without any singularities:

$$\int_a^b \left\{ r' A^{-\frac{2\gamma+1}{4}} H_{\frac{2\gamma+1}{4}}^{(2)}(kA'^{\gamma}) \frac{\partial v}{\partial n} - v(z) \frac{\partial}{\partial n} (r' A^{-\frac{2\gamma+1}{4}} H_{\frac{2\gamma+1}{4}}^{(2)}(kA'^{\gamma}) \right\} \times \\ \times f(z) \sqrt{1+f'^2(z)} dz = 0, \quad (1)$$

where  $A = (z - \eta)^2 + r^2$ ,  $a < \eta < b$ . The  $H$  are Hankel functions,  $\vec{n}$  is the unit vector directed along the surface normal to the outside,  $v_\gamma(z)$  are the harmonics of  $v(M)$  upon  $S$ . Thus,

$$v(M)|_S = \sum_{\gamma=-\infty}^{\infty} v_\gamma(z) e^{i\gamma\varphi}, \quad \frac{\partial v}{\partial n}|_S = \sum_{\gamma=-\infty}^{\infty} \frac{\partial v_\gamma}{\partial n}(z) e^{i\gamma\varphi}$$

When this steady case is to depend on time it is formally subjected to a Fourier transformation with the aid of the Kirchhoff-Sobolev formula, resulting in an integro-functional equation. An electromagnetic wave is dealt with also. This procedure is a translation of the results found for the scalar case into a vectorial analog. The field outside  $S$  is given in

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terms of the field strengths  $\vec{E}$  and  $\vec{H}$  upon  $S$  according to the Stratton-Ch'u formulas. It is pointed out that the equations always have solutions when the boundary conditions of the differential equations can be fulfilled. The uniqueness of the solutions obtained is demonstrated.

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